

A– Đạo hàm

1. Quy tắc tính đạo hàm

$$(c)' = 0$$

$$(u \times v)' = (u)'v + (v)'u$$

$$(u \pm v)' = (u)' \pm (v)'$$

$$(u_1 \pm u_2 \pm \cdots \pm u_n)' = (u_1)' \pm (u_2)' \pm \cdots (u_n)'$$

$$\left(\frac{u}{v}\right)' = \frac{(u)'v - (v)'u}{v^2}$$

$$\text{Nếu } y = y(u(x)) \text{ thì } y'(x) = y'(u) u'(x)$$

2. Công thức đạo hàm của hàm $f(x)$ với x là biến số

$$(kx)' = k$$

$$(x^n)' = nx^{n-1}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$(\cot x)' = -(1 + \cot^2 x) = -\frac{1}{\sin^2 x}$$

$$(e^x)' = e^x$$

$$(a^x)' = a^x \times \ln a$$

$$(\ln x)' = (\ln|x|)' = \frac{1}{x}$$

$$(\log_a x)' = (\log_a |x|)' = \frac{1}{x \times \ln a}$$

3. Đạo hàm của hàm hợp $f(u)$ với u là hàm số

$$(k \times u)' = k \times u'$$

$$(u^n)' = n \times u^{n-1} \times (u)'$$

$$\left(\frac{1}{u}\right)' = -\frac{(u)'}{u^2}$$

$$(\sqrt{u})' = -\frac{(u)'}{2\sqrt{u}}$$

$$(\sin u)' = \cos u \times (u)'$$

$$(\cos u)' = -\sin u \times (u)'$$

$$(\tan u)' = (1 + \tan^2 u) \times (u)' = \frac{(u)'}{\cos^2 u}$$

$$(\cot u)' = -(1 + \cot^2 u) \times (u)' = -\frac{(u)'}{\sin^2 x}$$

$$(e^u)' = e^u \times (u)'$$

$$(a^u)' = a^u \times \ln a \times (u)'$$

$$(\ln u)' = (\ln|u|)' = \frac{(u)'}{u}$$

$$(\log_a u)' = (\log_a |u|)' = \frac{(u)'}{u \times \ln a}$$

Mở rộng: Đạo hàm một số hàm phân thức hữu tỉ

$$y' = \left(\frac{ax + b}{cx + d} \right)' = \frac{ad - bc}{(cx + d)^2} = \frac{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{(cx + d)^2}$$

$$y' = \left(\frac{ax^2 + bx + c}{ex + f} \right)' = \frac{aex^2 + 2afx + (bf - ce)}{(ex + f)^2}$$

$$y' = \left(\frac{a_1x^2 + b_1x + c_1}{a_2x^2 + b_2x + c_2} \right)' = \frac{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}x^2 + \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}x + \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{(a_2x^2 + b_2x + c_2)^2}$$

B-Nguyên hàm – tích phân

1. Nguyên hàm của những hàm số sơ cấp thường gặp

$\int dx = x + C$	$\int du = u + C$
$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C \quad (\alpha \neq -1)$	$\int u^\alpha du = \frac{u^{\alpha+1}}{\alpha+1} + C \quad (\alpha \neq -1)$
$\int a^x dx = \frac{a^x}{\ln a} + C \quad (0 < a \neq 1)$	$\int a^u du = \frac{a^u}{\ln a} + C \quad (0 < a \neq 1)$
$\int \frac{dx}{x} = \ln x + C \quad (x \neq 0)$	$\int \frac{du}{u} = \ln u + C \quad (u \neq 0)$
$\int e^x dx = e^x + C$	$\int e^u du = e^u + C$
$\int \cos x dx = \sin x + C$	$\int \cos u du = \sin u + C$
$\int \sin x dx = -\cos x + C$	$\int \sin u du = -\cos u + C$
$\int \sin kx dx = -\frac{\cos kx}{k} + C$	$\int \cos kx dx = \frac{\sin kx}{k} + C$
$\int \frac{1}{\sin^2 x} dx = -\cot x + C$	$\int \frac{1}{\cos^2 u} du = \tan u + C$
$\int \frac{1}{\cos^2 x} dx = \tan x + C$	$\int \frac{1}{\sin^2 u} du = -\cot u + C$

2. Các nguyên hàm mở rộng

$$\int d(ax+b) = \frac{1}{a}(ax+b) + C$$

$$\int (ax+b)^\alpha dx = \frac{1}{a} \frac{(ax+b)^{\alpha+1}}{\alpha+1} + c, \alpha \neq -1$$

$$\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b| + c + c$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

$$\int a^{px+q} dx = \frac{1}{p \ln a} a^{px+q} + c$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2}) + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{|a|} + c$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \arccos \left| \frac{x}{a} \right| + c$$

$$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{x^2+a^2}}{x} \right| + c$$

$$\int \ln(ax+b) dx = \left(x + \frac{b}{a} \right) \ln(ax+b) - x + c$$

$$\int \sqrt{a^2-x^2} dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \arcsin \frac{x}{a} + c$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax} (a \sin bx - b \cos bx)}{a^2 + b^2} + c$$

$$\int e^{kx} dx = \frac{e^{kx}}{k} + C$$

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + c$$

$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + c$$

$$\int \operatorname{tg}(ax+b) dx = -\frac{1}{a} \ln |\cos(ax+b)| + c$$

$$\int \operatorname{cotg}(ax+b) dx = \frac{1}{a} \ln |\sin(ax+b)| + c$$

$$\int \frac{dx}{\sin^2(ax+b)} = -\frac{1}{a} \operatorname{cotg}(ax+b) + c$$

$$\int \frac{dx}{\cos^2(ax+b)} = \frac{1}{a} \operatorname{tg}(ax+b) + c$$

$$\int \arcsin \frac{x}{a} dx = x \arcsin \frac{x}{a} + \sqrt{a^2 - x^2} + c$$

$$\int \arccos \frac{x}{a} dx = x \arccos \frac{x}{a} - \sqrt{a^2 - x^2} + c$$

$$\int \operatorname{arctg} \frac{x}{a} dx = x \operatorname{arctg} \frac{x}{a} - \frac{a}{2} \ln(a^2 + x^2) + c$$

$$\int \operatorname{arc cotg} \frac{x}{a} dx = x \operatorname{arc cotg} \frac{x}{a} + \frac{a}{2} \ln(a^2 + x^2) + c$$

$$\int \frac{dx}{\sin(ax+b)} = \frac{1}{a} \ln \left| \operatorname{tg} \frac{ax+b}{2} \right| + c$$

$$\int \frac{dx}{\sin(ax+b)} = \frac{1}{a} \ln \left| \operatorname{tg} \frac{ax+b}{2} \right| + c$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax} (a \cos bx + b \sin bx)}{a^2 + b^2} + c$$